BC Calculus **Name** KEY

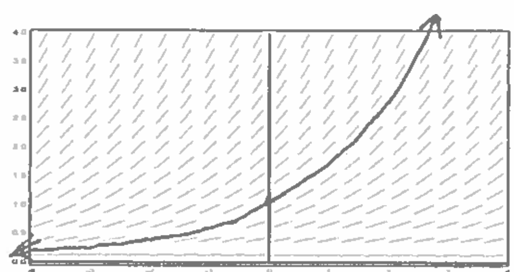
2020 Exam Practice

FR #6 (25 minutes: 15 points)

Set a timer for 25 minutes to complete this problem. You may use your notes, textbooks, or any materials I gave you throughout the year. You are not expected to use a calculator, but you may use one if you would like. You should show all your steps as if you did not have a calculator. I am guessing that the 25-minute problem will be worth 15 points and the 15-minute problem will be worth 10 points for a total of 25 points. The college board has said that the 25-minute problem will be worth 60% and the 15-minute problem will be worth 40%, so that is my best guess at how it may be broken down this year. Please show all appropriate mathematics: no bald answers!

Two scientists, Gill and Jeff, are studying the stinkbug population in the upstairs hallway. At the beginning of their study ($t = 0$) the stinkbug population is 1000. Both scientists decide to come up with differential equations that each believes will accurately model the population of stinkbugs. Gill decides on the model $\frac{dG}{dt} = \frac{1}{2}G$, with $G(0) = 1$, where t is measured in days and $G(t)$ is measured in thousands of stinkbugs. Jeff comes up with the alternate model $\frac{dJ}{dt} = \frac{1}{2}J(4 - J)$, with $J(0) = 1$, where t is measured in days and $J(t)$ is measured in thousands of stinkbugs.

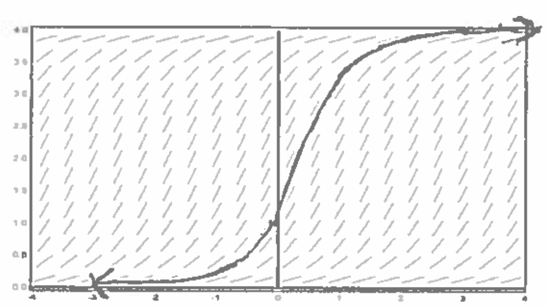
a) Scientist Gill draws his slope field as shown below. Draw in the particular solution that passes through the point (0,1). [2 points]



1 - starts at (0,1)

1 - correct shape

b) Scientist Jeff draws his slope field as shown below. Draw in the particular solution that passes through the point (0,1). [2 points]



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c) Find the particular solution to Scientist Gill's differential equation $\frac{dG}{dt} = \frac{1}{2}G$ with the initial condition that $G(0) = 1$. [5 points]

$\frac{1}{G}dG = \frac{1}{2}dt$

$\ln|G| = \frac{1}{2}t + c$

$G = e^{\frac{1}{2}t + c}$

$G = e^{\frac{1}{2}t + c}$

$1 = e^{0 + c}$

$C = \ln 1 = 0$

$G = e^{\frac{1}{2}t}$

OR

$G = Ce^{\frac{1}{2}t}$

$1 = Ce^{0}$

$C = 1$

$G = e^{\frac{1}{2}t}$

1 – separates var
1 – integrals
1 – +C
1 – plugs in (0,1)
1 – solution

d) Find the particular solution to Scientist Jeff's differential equation $\frac{dJ}{dt} = \frac{1}{2}J(4 - J)$ with the initial condition that $J(0) = 1$. (This solution will require the use of Partial Fractions decomposition.) [6 points]

$\frac{1}{J(4-J)}dJ = \frac{1}{2}dt$

$\frac{1}{J(4-J)} = \frac{A}{J} + \frac{B}{4-J}$

$1 = A(4-J) + BJ$

$J=4 \quad 1 = 4B \quad B = \frac{1}{4}$

$J=0 \quad 1 = 4A \quad A = \frac{1}{4}$

$\frac{1}{4}\int \frac{1}{J}dJ + \frac{1}{4}\int \frac{1}{4-J}dJ = \int \frac{1}{2}dt$

$\frac{1}{4}\ln|J| - \frac{1}{4}\ln|4-J| = \frac{1}{2}t + C_1$

$\ln|J| - \ln|4-J| = 2t + C$

$\ln\left|\frac{J}{4-J}\right| = 2t + C$

$\frac{J}{4-J} = Ce^{2t}$

$\frac{1}{4-1} = Ce^{0}$

$C = \frac{1}{3}$

$\frac{J}{4-J} = \frac{1}{3}e^{2t}$

$J = \frac{1}{3}e^{2t}(4-J)$

$J = \frac{4}{3}e^{2t} - \frac{1}{3}e^{2t}J$

$J + \frac{1}{3}e^{2t}J = \frac{4}{3}e^{2t}$

$J(1 + \frac{1}{3}e^{2t}) = \frac{4}{3}e^{2t}$

$J = \frac{\frac{4}{3}e^{2t}}{1 + \frac{1}{3}e^{2t}}$

OR

$J = \frac{4e^{2t}}{3 + e^{2t}}$

OR

$J = \frac{4}{3e^{-2t} + 1}$

1 – separates var
1 – int (left side)
1 – int (right side)
1 – +C
1 – Finds C
1 – solution