FRQ BC 1 2020 Solutions

x	0	3	5	9
$g(x)$	-2	-1	$-\frac{1}{8}$	$-\frac{1}{20}$
$g'(x)$	4	$\sqrt{8}$	$\sqrt{3}$	$\frac{3}{4}$

BC 1: The functions f and g are twice differentiable. Selected values of $g(x)$ and $g'(x)$ are given in the table above. The function f is defined by $f(x) = 2 + \int_0^{3x} g(t)dt$.

(A) Explain why there must be a number c, for $0 < c < 9$, such that $g'(c) = \frac{1}{3}$.

average rate: $\frac{g(3)-g(0)}{3-0} = \frac{(-1)-(-2)}{3} = \frac{1}{3}$ $\quad$ g is twice differentiable $\Rightarrow g$ is continuous

The Mean Value Theorem guarantees on $[0,3]$ which is within $[0,9]$ there is a value c such that $g'(c) = \frac{1}{3}$

(B) Evaluate $\int_0^3 40x f'''(x)dx$.

$$f'(x) = g(3x)(3) = 3g(3x) \Rightarrow f''(3x) = 9g'(3x)$$

$$\int x f'''(x)\ dx = x f''(x) - \int f''(x)\ dx = x f''(x) - f'(x)$$

$$u = x \Rightarrow du = 1$$
$$dv = f'''(x)dx \Rightarrow v = f''(x)$$

$$40\int_0^3 x f'''(x)\ dx = 40\left[x f''(x) - f'(x)\right]_0^3 = 40\left[3f''(3) - f'(3)\right] - 40\left[(0)f''(0) - f'(0)\right]$$

$$= 40\left[27g'(9) - 3g(9)\right] - 40\left[-3g(0)\right] = 40\left[27\left(\frac{3}{4}\right) - 3\left(-\frac{1}{20}\right)\right] - 40\left[-3(-2)\right]$$

$$= 40\left[\left(\frac{81}{4}\right) + \left(\frac{3}{20}\right)\right] - 40[6] = \left[40\left(\frac{81}{4}\right) + 40\left(\frac{3}{20}\right)\right] - 40[6] = 810 + 6 - 240 = 576$$

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(C) Using a right Riemann sum with three subintervals indicated in the table above, approximate the length of the curve of $g(x)$ from $x = 0$ to $x = 9$.

$$L = \int_0^9 \sqrt{1+\left(g'(x)\right)^2} \approx 3\sqrt{1+\left(g'(3)\right)^2} + 2\sqrt{1+\left(g'(5)\right)^2} + 4\sqrt{1+\left(g'(9)\right)^2}$$

$$= 3\sqrt{1+\left(\sqrt{8}\right)^2} + 2\sqrt{1+\left(\sqrt{3}\right)^2} + 4\sqrt{1+\left(\frac{3}{4}\right)^2} = 3\sqrt{9} + 2\sqrt{4} + 4\sqrt{\frac{25}{16}} = 9+4+5 = 18$$

(D) Let $P_n(x)$ denote the nth degree Taylor polynomial for f about $x = 0$. Find $P_2(x)$.

$$f(x) = 2 + \int_0^{3x} g(t)dt \Rightarrow f'(x) = 3g(3x) \Rightarrow f''(x) = 9g'(3x)$$

$$f(0) = 2 \Rightarrow f'(0) = 3(-2) = -6 \Rightarrow f''(x) = 9(4) = 36$$

$$P_2(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 = 2 - 6x + 18x^2$$

(E) Consider the geometric series $\sum_{n=0}^{\infty} ar^n$ whose first three terms are defined by the polynomial $P_2(x)$ found in part (D). Find the sum of this series when $x = \frac{1}{6}$, or show that the series diverges.

$$\sum_{n=0}^{\infty} ar^n = 2 - 6x + 18x^2 \Rightarrow a = 2, r = \frac{-6x}{2} = \frac{18x^2}{-6x} = -3x$$

$$x = \frac{1}{6} \Rightarrow \sum_{n=0}^{\infty} 2(-3x)^n = \sum_{n=0}^{\infty} 2\left(-3\left(\frac{1}{6}\right)\right)^n = \sum_{n=0}^{\infty} 2\left(\frac{-1}{2}\right)^n = \frac{2}{1-\left(-\frac{1}{2}\right)} = \frac{2}{3/2} = \frac{4}{3}$$