BC Calculus
2020 Exam Practice
FR #4 (25 minutes: 15 points)

Name KEY

Set a timer for 25 minutes to complete this problem. You may use your notes, textbooks, or any materials I gave you throughout the year. You are not expected to use a calculator, but you may use one if you would like. You should show all your steps as if you did not have a calculator. I am guessing that the 25-minute problem will be worth 15 points and the 15-minute problem will be worth 10 points for a total of 25 points. The college board has said that the 25-minute problem will be worth 60% and the 15-minute problem will be worth 40%, so that is my best guess at how it may be broken down this year. Please show all appropriate mathematics: no bald answers!

Consider the function $f(x) = 2\ln(x-3)$, which passes through the point (4,0).

a) Find the equation of the line tangent to $f(x)$ at the point (4,0). Use that equation to find an approximation for $f(4.2)$. [3 points]

$f'(x) = \frac{2}{x-3}$

$f'(4) = \frac{2}{4-3} = 2$

$f(4) = 0$

$y = 2(x-4)$

$f(4.2) \approx 2(4.2-4)$ or $f(4.2) \approx 0.4$

1 - $f'(4) = 2$
1 - tan line
1 - approx

b) Is the approximation you found in part (a) greater than or less than the actual value of $f(4.2)$? Justify your answer. [2 points]

$f'(x) = 2(x-3)^{-1}$

$f''(x) = -2(x-3)^{-2}$

$f''(x) = \frac{-2}{(x-3)^2}$

On $[4, 4.2]$, $f''(x) < 0$ so $f(x)$ is concave down on that interval. Therefore, the tangent line is above the curve. Greater than

1 - finds $f''(x)$
1 - Greater than with reason

c) Starting at the point (4,0), use Euler's Method to approximate the value of $f(4.2)$ using two steps of equal size. [4 points]

$f'(x) = \frac{2}{x-3}$ $f'(4) = 2$ $y = 2(x-4)$

plug in $x = 4.1$ $y = 2(4.1-4) = 0.2$ $m = \frac{2}{4.1-3} = \frac{2}{1.1}$

$y = \frac{2}{1.1}(x-4.1) + 0.2$

plug in $x = 4.2$ $y = \frac{2}{1.1}(4.2-4.1)+0.2 \approx f(4.2)$

or $f(4.2) \approx 0.382$

1 - first tangent line
1 - second slope
1 - second tangent line
1 - approx for $f(4.2)$

Consider the function $f(x) = 2\ln(x-3)$, which passes through the point (4,0).

d) **Find the third-degree Taylor Polynomial for $f(x)$, centered at $x = 4$. Use the polynomial to approximate the value of $f(4.2)$. [4 points]**

$f(x) = 2\ln(x-3)$ $\quad f(4) = 0$

$f'(x) = \frac{2}{x-3}$ $\quad f'(4) = 2$

$f''(x) = \frac{-2}{(x-3)^2}$ $\quad f''(4) = -2$

$f'''(x) = \frac{4}{(x-3)^3}$ $\quad f'''(4) = 4$

$P_3(x) = 2(x-4) - \frac{2}{2!}(x-4)^2 + \frac{4}{3!}(x-4)^3$

OR

$P_3(x) = 2(x-4) - (x-4)^2 + \frac{2}{3}(x-4)^3$

$f(4.2) \approx 2(4.2-4) - (4.2-4)^2 + \frac{2}{3}(4.2-4)^3$

OR

$f(4.2) \approx 0.365$

- 1 - linear term
- 1 - quad term
- 1 - cubic term
- 1 - approx

e) **Show the setup of a definite integral that will yield the length of the curve of $f(x)$ from $x = 4$ to $x = 4.2$. You DO NOT have to evaluate the integral. [2 points]**

$$\int_4^{4.2} \sqrt{1 + \left(\frac{2}{x-3}\right)^2}\, dx$$

OR

$$\int_4^{4.2} \sqrt{1 + \frac{4}{(x-3)^2}}\, dx$$

- 1 - limits w/ integral
- 1 - integrand