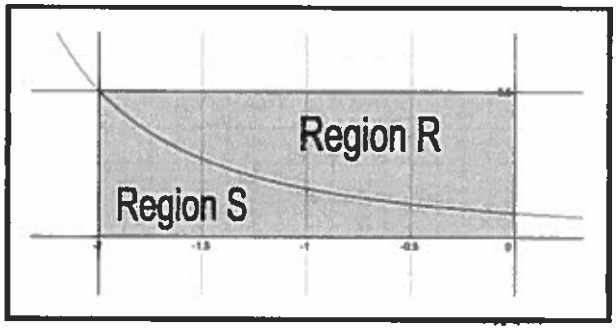
BC Calculus **Name** KEY
2020 Exam Practice
FR #3 (25 minutes: 15 points)

Set a timer for 25 minutes to complete this problem. You may use your notes, textbooks, or any materials I gave you throughout the year. You are not expected to use a calculator, but you may use one if you would like. You should show all your steps as if you did not have a calculator. I am guessing that the 25-minute problem will be worth 15 points and the 15-minute problem will be worth 10 points for a total of 25 points. The college board has said that the 25-minute problem will be worth 60% and the 15-minute problem will be worth 40%, so that is my best guess at how it may be broken down this year. Please show all appropriate mathematics: no bald answers!

As shown in the graph below, Region R is bounded by the function $f(x) = \frac{1}{x^2+7x+12}$, the horizontal line $y = 0.5$, and the vertical line $x = 0$. Region S is bounded by $f(x) = \frac{1}{x^2+7x+12}$, the horizontal line $y = 0$, and the vertical lines $x = -2$ and $x = 0$.



a) Using the method of Partial Fractions, set up and evaluate a definite integral to find the area of Region S. [3 points]

$\int_{-2}^{0} \frac{1}{x^2+7x+12}\,dx$ $\quad \frac{1}{x^2+7x+12} = \frac{A}{x+3} + \frac{B}{x+4}$

$1 = A(x+4) + B(x+3)$

$x = -4 \quad B = -1$

$x = -3 \quad A = 1$

$\int_{-2}^{0} \left(\frac{1}{x+3} - \frac{1}{x+4}\right)dx$

$\left[\ln|x+3| - \ln|x+4|\right]_{-2}^{0}$

$(\ln 3 - \ln 4) - (\ln 1 - \ln 2)$

OR

$\ln 3 - \ln 4 + \ln 2$

OR

$\ln \frac{3}{2}$

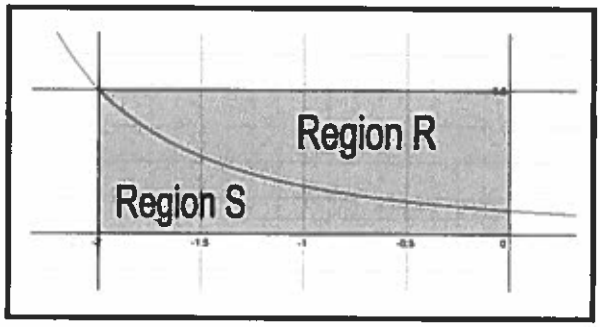
1 - correct partial fractions
1 - correct integral
1 - correct area

b) Show the setup of an integral that could be used to find the area of Region R. You DO NOT need to evaluate the integral. [2 points]

$\int_{-2}^{0} \left(0.5 - \frac{1}{x^2+7x+12}\right)dx$

1 - limits
1 - integrand

As shown in the graph below, Region R is bounded by the function $f(x) = \frac{1}{x^2+7x+12}$, the horizontal line $y = 0.5$, and the vertical line $x = 0$. Region S is bounded by $f(x) = \frac{1}{x^2+7x+12}$, the horizontal line $y = 0$, and the vertical lines $x = -2$ and $x = 0$.



c) Show the setup of the integral that would yield the volume of the solid formed when Region S is rotated about the x-axis. You DO NOT need to evaluate the integral. [2 points]

$$\pi \int_{-2}^{0} \left(\frac{1}{x^2+7x+12} \right)^2 dx$$

1 - π, limits
1 - integrand

d) Region R is the base of a solid with cross-sections perpendicular to the x-axis that are squares. Show the setup of the integral that would yield the volume of the solid. You DO NOT need to evaluate the integral. [2 points]

$$\int_{-2}^{0} \left(0.5 - \frac{1}{x^2+7x+12} \right)^2 dx$$

1 - limits, NO constant
1 - integrand

e) Show the setup of the integral that would yield the volume of the solid formed when Region R is rotated about the y-axis. You DO NOT need to evaluate the integral. [2 points]

Shell Method

$$2\pi \int_{-2}^{0} -x \left(0.5 - \frac{1}{x^2+7x+12} \right) dx$$

1 - 2π, limits
1 - integrand

f) Write an expression including an integral that would yield the perimeter of Region S. You DO NOT have to evaluate the integral portion of the expression. [4 points]

Perimeter = P

$f(0) = \frac{1}{12}$

$f(x) = (x^2+7x+12)^{-1}$

$f'(x) = -(x^2+7x+12)^{-2}(2x+7)$

$f'(x) = \frac{-(2x+7)}{(x^2+7x+12)^2}$

$$P = 0.5 + 2 + \frac{1}{12} + \int_{-2}^{0} \sqrt{1 + \left(\frac{-(2x+7)}{(x^2+7x+12)^2} \right)^2} \, dx$$

1 - 0.5 + 2
1 - $\frac{1}{12}$
1 - limits
1 - integrand