BC Calculus **Name** KEY

2020 Exam Practice

FR #2 (25 minutes: 15 points)

Set a timer for 25 minutes to complete this problem. You may use your notes, textbooks, or any materials I gave you throughout the year. You are not expected to use a calculator, but you may use one if you would like. You should show all your steps as if you did not have a calculator. I am guessing that the 25-minute problem will be worth 15 points and the 15-minute problem will be worth 10 points for a total of 25 points. The college board has said that the 25-minute problem will be worth 60% and the 15-minute problem will be worth 40%, so that is my best guess at how it may be broken down this year. Please show all appropriate mathematics: no bald answers!

The function $f(x)$ is given by this accumulation function:

$$f(x) = \int_2^x t \cdot e^t dt$$

a) Find all intervals on which $f(x)$ is increasing or decreasing. Give reasons for your answers. [3 points]

$f'(x) = xe^x = 0$

$x = 0$

$f'(x)$: $-$ | $+$ at 0

INC $(0, \infty)$ b/c $f'(x) > 0$

DEC $(-\infty, 0)$ b/c $f'(x) < 0$

1 - finds $f'(x)$
1 - INC/Reason
1 - DEC/Reason

b) Find all intervals on which $f(x)$ is concave up and concave down. Give reasons for your answer. [3 points]

$f''(x) = xe^x + e^x = 0$

$e^x(x+1) = 0$

$x = -1$

$f''(x)$: $-$ | $+$ at -1

CU $(-1, \infty)$ b/c $f''(x) > 0$

CD $(-\infty, -1)$ b/c $f''(x) < 0$

1 - Correct $f''(x)$
1 - CU/Reason
1 - CD/Reason

c) Find $f(x)$ by completing the definite integral. Show all steps. [3 points]

$\int_2^x te^t dt$ (Int. by Parts)

	u	dv
+	t	$e^t dt$
−	1	e^t
+	0	e^t

$\left[te^t - e^t\right]_2^x$

$(xe^x - e^x) - (2e^2 - e^2)$

$f(x) = xe^x - e^x - e^2$

1 - uses int by parts
1 - correct integral
1 - correct substitution to find $f(x)$

The function $f(x)$ is given by this accumulation function:

$$f(x) = \int_2^x t \cdot e^t dt$$

d) On the interval [-1,4], find the absolute maximum and absolute minimum values of $f(x)$. Justify your answers. [3 points]

$f'(x) = xe^x = 0$

$x = 0$ ← Critical number

Candidates are $x = -1, 0, 4$

(Endpoints and critical #s)

x	$f(x)$
-1	$-e^{-1} - e^{-1} - e^2 = -\frac{2}{e} - e^2$ ← Abs. Min.
0	$0 - e^0 - e^2 = -1 - e^2$
4	$4e^4 - e^4 - e^2 = 3e^4 - e^2$ ← Abs. Max

I found these values by plugging into the function in part C.

- 1 - discusses candidates
- 1 - Abs Max
- 1 - Abs. Min

e) Find the equation of the line tangent to $f(x)$ at the point when $x = 2$. Use the equation of the tangent line to approximate the value of $f(4)$. [3 points]

$f'(x) = xe^x$

$f'(2) = 2e^2$

$f(2) = \int_2^2 te^t dt = 0$

$y = 2e^2(x-2)$

$f(4) \approx 2e^2(4-2)$

OR

$f(4) \approx 4e^2$

- 1 - finds slope
- 1 - tangent line
- 1 - approx. for $f(4)$