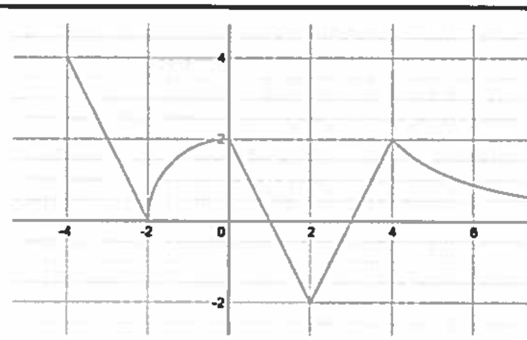
BC Calculus **Name** KEY
2020 Exam Practice
FR #1 (25 minutes: 15 points)

Set a timer for 25 minutes to complete this problem. You may use your notes, textbooks, or any materials I gave you throughout the year. You are not expected to use a calculator, but you may use one if you would like. You should show all your steps as if you did not have a calculator. I am guessing that the 25-minute problem will be worth 15 points and the 15-minute problem will be worth 10 points for a total of 25 points. The college board has said that the 25-minute problem will be worth 60% and the 15-minute problem will be worth 40%, so that is my best guess at how it may be broken down this year. Please show all appropriate mathematics: no bald answers!

The graph of f', consisting of 3 line segments, a quarter circle, and a portion of the graph of $y = \frac{32}{x^2}$, is shown below. It is known that $f(0) = 5$.



Graph of f'

a) On the interval [-4,6], find all x-values at which $f(x)$ has relative maxima and relative minima. Give a reason for your answers. [3 points]

$f'(x) = 0$

$x = 1, 3$

$x = 1$ is a Rel Max b/c f' changes from positive to negative at that place.

$x = 3$ is a Rel Min b/c f' changes from negative to positive at that place.

1 - finds $x = 1, 3$
1 - Max/Reason
1 - Min/Reason

b) On the interval [-4,6], find all x-values at which $f(x)$ has points of inflection. Give a reason for your answer. [3 points]

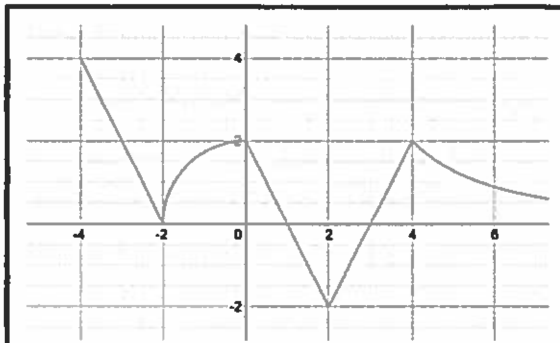
$f''(x)$ is undefined at $x = -2, 0, 2, 4$

$x = -2, 0, 2, 4$ b/c f'' changes sign at those places.

OR b/c f' goes from dec to inc or from inc to dec at those places

1 - gets at least one value
1 - gets all the others, with no extras
1 - Reason

The graph of f', consisting of 3 line segments, a quarter circle, and a portion of the graph of $y = \frac{32}{x^2}$, is shown below. It is known that $f(0) = 5$.



Graph of f'

c) Write an expression for $f(x)$ that includes an integral. Use that expression to find the values of $f(1)$ and $f(3)$. [3 points]

$$f(x) = 5 + \int_0^x f'(t)\,dt$$

$$f(1) = 5 + \int_0^1 f'(t)\,dt = 5 + \tfrac{1}{2}(1)(2) = 6$$

$$f(3) = 5 + \int_0^3 f'(t)\,dt = 5 + \tfrac{1}{2}(1)(2) - \tfrac{1}{2}(2)(2) = 4$$

1 – $f(x)$
1 – correct $f(1)$
1 – correct $f(3)$

d) On the interval [-4,6], find the absolute maximum and absolute minimum values of $f(x)$. Justify your answers. [3 points]

$f'(x)$: + + − + on -4, -2, 1, 3, 6

Only candidates are when $f'(x) = 0$ and the endpoints. $x = -2$ is not a candidate, b/c $f'(x)$ doesn't change sign

$x = -4, -2, 1, 3, 6$

x	$f(x)$
-4	$1-\pi$
1	6
3	4
6	$7\frac{2}{3}$

Abs. Min.

$$5 + \int_0^{-4} f'(t) = 5 - \tfrac{1}{4}\pi(2)^2 - \tfrac{1}{2}(2)(4) = 1 - \pi$$

Abs Max

$$5 + \int_0^6 f'(t) = 5 + \int_4^6 \frac{32}{x^2}\,dx$$

$$5 - \left[\frac{32}{x}\right]_4^6 = 5 - \left[\frac{32}{6} - \frac{32}{4}\right] = 5 - \left[\frac{16}{3} - 8\right] = 5 - \left(-\frac{8}{3}\right) = 7\frac{2}{3}$$

1 – candidates
1 – min
1 – max

e) Find the area of the region bounded by the x-axis, the vertical line $x = 4$, and the portion of the graph of $y = \frac{32}{x^2}$. (Note that there is no upper bound, so this will be an improper integral.) [3 points]

$$\int_4^\infty \frac{32}{x^2}\,dx = \lim_{b\to\infty} \int_4^b \frac{32}{x^2}\,dx = \lim_{b\to\infty} \left[\frac{-32}{x}\right]_4^b$$

$$= \lim_{b\to\infty} \left[\frac{-32}{b} + \frac{32}{4}\right] = \lim_{b\to\infty} \left[8 - \frac{32}{b}\right] = 8$$

1 – setup of integral
1 – correct limit exp. and integral
1 – answer